Effects of the ground state correlations on the structure of vibrational states

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1 Introduction.

The Random Phase Approximation (RPA) is the basic method to describe the nuclear vibrational motion. It is well known that the RPA violates the Pauli principle and many attempts have been performed to improve it ([1–15]). There is a rather complete list of references on that subject in paper [10]. Renormalized RPA equations that include corrections for the ground state correlations (GSC), have been applied not only to study properties of the low-lying isoscalar vibrations in spherical nuclei [11,12], but also to investigate the charge-exchange modes in nuclei [16,17] and the giant resonances in metal clusters [18]. To describe excited states all above mentioned works use the one phonon states that can be created acting by the relevant collective operator on the ground state of the system, which is treated as the vacuum state. From another point of view it is well known that due to the anharmonicity of vibrations there is a coupling between one-phonon and more complex states [19,20]. Taking into account such a coupling it is possible to describe particularities

of the low-lying states and damping of the giant resonances [20]. Up to now such coupling was considered for the RPA phonons only [20].

In the present paper we use phonons of the extended RPA (ERPA) [12] as a basis on which the quasi-particle phonon model (QPM) [20] equations are generalized so as to account for the GSC in the description of nuclear vibrational states constructed by one- and two-phonon configurations. Besides the GSC, we take into account the Pauli principle corrections arising in the two-phonon terms due to the fermion structure of the phonon operators. As an example we study the effect of the GSC on the energies, transition probabilities and transition densities of the low-lying vibrational states in ^{68}Zn and compare present results with the results within other approaches.

2. Basic formulae

We employ the QPM - Hamiltonian including an average nuclear field described as the Woods-Saxon potential, pairing interactions, the isoscalar and isovector particle–hole (p–h) residual forces in separable form with the Bohr–Mottelson radial dependence [19]:

$$
H = \sum_{\tau} \{ \sum_{jm} (E_j - \lambda_\tau) a_{jm}^\dagger a_{jm} - \frac{1}{4} G_\tau^{(0)} : (P_0^\dagger P_0)^\tau : -
$$
$$
\frac{1}{2} \sum_{\lambda\mu\sigma=\pm 1} (\kappa_0^{(\lambda)} + \sigma \kappa_1^{(\lambda)}) : (M_{\lambda\mu}^\dagger(\tau) M_{\lambda\mu}(\sigma\tau)) : \}
\tag{1}
$$

We sum over the proton(p) and neutron(n) indexes and the notation $\{\tau = (n,p)\}$ is used and a change $\tau \leftrightarrow -\tau$ means a change $p \leftrightarrow n$. The single-particle states are specified by the quantum numbers (jm); E_j are the single-particle energies; λ_τ is the chemical potential; $G_\tau^{(0)}$ and $\kappa^{(\lambda)}$ are the strengths in the p–p and in the p–h channel, respectively. The pair creation and the multipole operators entering the normal products in (1) are defined as follows:

$$
P_0^+ = \sum_{jm} (-1)^{j-m} a_{jm}^+ a_{j-m}^+
\tag{2}
$$

$$
M_{\lambda\mu}^+(\tau) = \frac{1}{\sqrt{2\lambda+1}} \sum_{jj'mm'}^{(\tau)} (-1)^{j+m} \langle jmj'-m' \mid \lambda\mu \rangle f_{j'j}^{(\lambda)} a_{jm}^+ a_{j'm'}^+
\tag{3}
$$

where $f_{jj'}^{(\lambda)}$ are the single particle radial matrix elements of residual forces.

In what follows we work in quasiparticle (qp) representation, defined by the canonical Bogoliubov transformation:

$$
a_{jm}^+ = u_j \alpha_{jm}^+ + (-1)^{j-m} v_j \alpha_{j-m}
\tag{4}
$$

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The Hamiltonian can be represented in terms of bifermion quasiparticle operators (and their conjugate ones):

$$B(jj'; \lambda\mu) = \sum_{mm'} (-1)^{j'+m'} \langle jmj'm' \mid \lambda\mu \rangle \alpha_{jm}^+ \alpha_{j'-m'} \tag{5}$$

$$A^+(jj'; \lambda\mu) = \sum_{mm'} \langle jmj'm' \mid \lambda\mu \rangle \alpha_{jm}^+ \alpha_{j'm'}^+ \tag{6}$$

The phonon creation operators are defined in the 2-qp space in a standard fashion:

$$Q_{\lambda\mu,i}^+ = \frac{1}{2} \sum_{jj'} \{ \psi_{jj'}^{\lambda i} A^+(jj'; \lambda\mu) - (-1)^{\lambda-\mu} \varphi_{jj'}^{\lambda i} A(jj'; \lambda - \mu) \} \tag{7}$$

where the index $\lambda = 0, 1, 2, 3, ...$ denotes multipolarity and μ is its z-projection in the laboratory system. The following relation can be proved using the exact commutators of the fermion operators:

$$\langle 0 \mid [Q_{\lambda\mu,i}, Q_{\lambda'\mu',i'}^+] \mid 0 \rangle = \frac{1}{2} \delta_{\lambda\lambda'} \delta_{\mu\mu'} \sum_{jj'} (1 - q_{jj'}) [\psi_{jj'}^{\lambda i} \psi_{jj'}^{\lambda i'} - \varphi_{jj'}^{\lambda i} \varphi_{jj'}^{\lambda i'}] \tag{8}$$

where $\mid 0 \rangle$ is the phonon vacuum, $q_{jj'} = q_j + q_{j'}$ and q_j is the quasiparticle distribution in the ground state: $q_j \equiv (2j+1)^{-\frac{1}{2}} \langle 0 \mid B(jj; 00) \mid 0 \rangle$.

The phonon and the pairing characteristics are determined by the following non–linear system of equations:

$$\sum_{\tau} \left[\left(k_0^{(\lambda)} + k_1^{(\lambda)} \right) X_\tau^{\lambda i} - 2k_0^{(\lambda)} k_1^{(\lambda)} X_\tau^{\lambda i} X_{-\tau}^{\lambda i} \right] = 1 \tag{9}$$

$$X_\tau^{\lambda i} = \sum_{jj'}^{(\tau)} \frac{(f_{jj'}^{(\lambda)} u_{jj'}^{(+)})^2 \varepsilon_{jj'} (1 - q_{jj'})}{\varepsilon_{jj'}^2 - \omega_{\lambda i}^2} \tag{10}$$

$$\sum_{jj'} (1 - q_{jj'}) [(\psi_{jj'}^{\lambda i})^2 - (\varphi_{jj'}^{\lambda i})^2] - 2 = 0 \tag{11}$$

$$\frac{G_\tau^{(0)}}{2} \sum_j^{(\tau)} \frac{(j + 1/2)(1 - 2q_j)}{\sqrt{\Delta_\tau^2 + (E_j - \lambda_\tau)^2}} = 1 \tag{12}$$

$$\sum_j^{(\tau)} (j + 1/2) \left[1 - \frac{(E_j - \lambda_\tau)(1 - 2q_j)}{\sqrt{\Delta_\tau^2 + (E_j - \lambda_\tau)^2}} \right] = N^{(\tau)} \tag{13}$$

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$$q_j = \frac{1}{2} \sum_{\lambda i, j'} \frac{2\lambda + 1}{2j + 1} (1 - q_{jj'}) (\varphi_{jj'}^{\lambda i})^2 \tag{14}$$

The formulae for the quasiparticle energies $\varepsilon_j = \sqrt{\Delta_\tau^2 + (E_j - \lambda_\tau)^2}$ and for the coefficients u_j, v_j remain the same as in the usual BCS theory; the new values for $\Delta_\tau \equiv \frac{1}{2} G_\tau^{(0)} \sum_j^{(\tau)} (1 - 2q_j)(2j+1) u_j v_j$ and λ_τ, come from the eqs. (12) and (13); $\varepsilon_{jj'} = \varepsilon_j + \varepsilon_{j'}$, $u_{jj'}^{(+)} = u_j v_{j'} + v_j u_{j'}$. The pairing vibrations ($\lambda = 0$) have been considered in [23].

The system of nonlinear equations (9)-(14) includes effects of the isoscalar and isovector forces and it is a generalization of equations derived in [1,11,12,22]. This system treats the GSC self-consistently and describes the coupling between different vibrations, different phonon roots of a certain multipolarity and the pairing. The present scheme is called Extended RPA (ERPA). The factors $(1 - q_{jj'})$, distinguishing the new equations from the conventional BCS and RPA ones, take into account the blocking effect due to the Pauli principle. If we put $q_{jj'} = 0$ in the r.h.s. of eq. (14), we get the expression for the quasiparticle distribution in the ground state in the RPA case [3,9].

The GSC affect not only the RPA, but they also should change the quasiparticle-phonon coupling. To take into account such effects we follow the basic ideas of the QPM. Hereafter we derive the generalized QPM equations which take into account the GSC beyond the RPA. As it was shown in our previous paper [23] the pairing vibrations give a negligible contribution to q_j. On the other hand the two-phonon configurations including the pairing vibration phonons have an energy essentially higher than the configurations constructed from usual vibration phonons. That is why we do not take into account the coupling with the pairing vibrations in what follows. Using the completeness and orthogonality conditions for the phonon operators one can express bifermion operators A^+ and A by phonons:

$$A^+(jj'; \lambda\mu) + (-)^{\lambda-\mu} A(jj'; \lambda - \mu) =$$
$$(1 - q_{jj'}) \sum_i (\psi_{jj'}^{\lambda i} + \phi_{jj'}^{\lambda i})(Q_{\lambda\mu i}^+ + (-)^{\lambda-\mu} Q_{\lambda-\mu i}) \tag{15}$$

The initial Hamiltonian (1) can be rewritten in terms of quasiparticle and phonon operators in following form:

$$H = h_0 + h_{pp} + h_{QQ} + h_{QB} \tag{16}$$

$$h_0 + h_{pp} = \sum_{jm} \varepsilon_j \alpha_{jm}^+ \alpha_{jm} \tag{17}$$

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$$h_{QQ} = -\frac{1}{8} \sum_{\lambda\mu ii'\tau} \frac{X_\tau^{\lambda i} + X_\tau^{\lambda i'}}{\sqrt{\mathcal{Y}_\tau^{\lambda i} \mathcal{Y}_\tau^{\lambda i'}}} (Q_{\lambda\mu i}^+ + (-)^{\lambda-\mu} Q_{\lambda-\mu i})(Q_{\lambda-\mu i'}^+ + (-)^{\lambda+\mu} Q_{\lambda\mu i'}) \quad (18)$$

$$h_{QB} = -\frac{1}{2\sqrt{2}} \sum_{\lambda\mu ijj'\tau} \frac{v_{jj'}^{(-)} f_{jj'}^{(\lambda)}}{\sqrt{\mathcal{Y}_\tau^{\lambda i}}} ((-)^{\lambda-\mu} Q_{\lambda\mu i}^+ + Q_{\lambda-\mu i}) B_\tau(jj'; \lambda - \mu) + h.c. \quad (19)$$

where $v_{jj'}^{(-)} = u_j u_{j'} - v_j v_{j'}$ and

$$\mathcal{Y}_\tau^{\lambda i} = Y_\tau^{\lambda i} + Y_{-\tau}^{\lambda i} \left\{ \frac{1 - \left(k_0^{(\lambda)} + k_1^{(\lambda)}\right) X_\tau^{\lambda i}}{\left(k_0^{(\lambda)} - k_1^{(\lambda)}\right) X_{-\tau}^{\lambda i}} \right\}^2 \quad (20)$$

$$Y_\tau^{\lambda i} = \sum_{jj'}^{(\tau)} \frac{(f_{jj'}^{(\lambda)} u_{jj'}^{(+)})^2 \, \varepsilon_{jj'} \, \omega_{\lambda i}(1 - q_{jj'})}{[\varepsilon_{jj'}^2 - \omega_{\lambda i}^2]^2} \quad (21)$$

One can prove that the solutions of the system of equations (9)–(14) obey the following equality:

$$\langle Q_{\lambda\mu i} \mid H \mid Q_{\lambda\mu i}^+ \rangle = \omega_{\lambda i} \quad (22)$$

The term h_{QB} is responsible for the mixing of the configurations and, therefore, for the description of many characteristics of the excited states of even–even nuclei. In the simplest case the wave functions of those states could be written down as:

$$\Psi_\nu(\lambda\mu) = \{\sum_i R_i(\lambda\nu) Q_{\lambda\mu i}^+ + \sum_{\lambda_1 i_1 \lambda_2 i_2} P_{\lambda_1 i_1}^{\lambda_2 i_2}(\lambda\nu) \left[Q_{\lambda_1\mu_1 i_1}^+ Q_{\lambda_2\mu_2 i_2}^+ \right]_{\lambda\mu} \}|0\rangle \quad (23)$$

with the normalization condition:

$$\langle \Psi_\nu(JM) \mid \Psi_\nu(JM) \rangle = \sum_i R_i^2(J\nu) +$$

$$2 \sum_{\lambda_1 i_1 \lambda_2 i_2} (P_{\lambda_2 i_2}^{\lambda_1 i_1}(J\nu))^2 (1 + K^J(\lambda_1 i_1, \lambda_2 i_2)) = 1 \quad (24)$$

where

$$K^J(\lambda_1 i_1, \lambda_2 i_2) \equiv K^J(\lambda_1 i_1, \lambda_2 i_2 \mid \lambda_2 i_2, \lambda_1 i_1)$$

and

$$K^J(\lambda_2 i_2, \lambda i^{'} \mid \lambda i, \lambda_2 i_2) = (2\lambda + 1)(2\lambda_2 + 1)\frac{1}{1 + \delta_{i,i'}\delta_{\lambda i, \lambda_2 i_2}} \times$$

$$\sum_{j_1 j_2 j_3 j_4} (1 - \frac{1}{2}q_{j_1 j_2 j_3 j_4})(-1)^{j_2 + j_4 + J} \begin{Bmatrix} j_1 & j_2 & \lambda_2 \\ j_4 & j_3 & \lambda \\ \lambda & \lambda_2 & J \end{Bmatrix} \times$$

$$(\psi_{j_1 j_4}^{\lambda i'}\psi_{j_3 j_4}^{\lambda i}\psi_{j_3 j_2}^{\lambda_2 i_2}\psi_{j_1 j_2}^{\lambda_2 i_2} - \varphi_{j_3 j_2}^{\lambda_2 i_2}\varphi_{j_1 j_2}^{\lambda_2 i_2}\varphi_{j_3 j_4}^{\lambda i'}\varphi_{j_1 j_4}^{\lambda i}) \qquad (25)$$

$$q_{j_1 j_2 j_3 j_4} \equiv q_{j_1} + q_{j_2} + q_{j_3} + q_{j_4}$$

The mean value of H is

$$\langle \Psi_\nu(JM) \mid H \mid \Psi_\nu(JM) \rangle = \sum_i R_i^2(J\nu)\omega_{Ji} + 2\sum_{\lambda_1 i_1 \lambda_2 i_2} (P_{\lambda_2 i_2}^{\lambda_1 i_1}(J\nu))^2 \times$$

$$(\omega_{\lambda_1 i_1} + \omega_{\lambda_2 i_2} + \Delta\omega^J(\lambda_1 i_1, \lambda_2 i_2))(1 + K^J(\lambda_1 i_1, \lambda_2 i_2)) +$$
$$2\sum_{\lambda_1 i_1, i\lambda_2 i_2} R_i(J\nu)P_{\lambda_2 i_2}^{\lambda_1 i_1}(J\nu)U_{\lambda_2 i_2}^{\lambda_1 i_1}(J\nu)(1 + K^J(\lambda_1 i_1, \lambda_2 i_2)) \quad (26)$$

where the matrix element coupling one- and two-phonon configurations is:

$$\langle Q_{J\nu}|h_{QB}| \left[Q_{\lambda_1 i_1}^+ Q_{\lambda_2 i_2}^+\right]_J \rangle = U_{\lambda_2 i_2}^{\lambda_1 i_1}(J\nu)(1 + K^J(\lambda_1 i_1, \lambda_2 i_2)) \qquad (27)$$

$$U_{\lambda_2 i_2}^{\lambda_1 i_1}(\lambda i) = (-1)^{\lambda_1 + \lambda_2 + \lambda}U_{\lambda_1 i_1}^{\lambda_2 i_2}(\lambda i)$$

$$U_{\lambda_2 i_2}^{\lambda_1 i_1}(\lambda i) \equiv \sum_\tau U_{\lambda_2 i_2}^{\lambda_1 i_1}(\lambda i, \tau)$$

$$U_{\lambda_2 i_2}^{\lambda_1 i_1}(\lambda i, \tau) = \frac{1}{\sqrt{2}}\sqrt{(2\lambda_1 + 1)(2\lambda_2 + 1)} \sum_{j_1 j_2 j_3}^{(\tau)} (1 - q_{j_2 j_3}) \times$$

$$(\frac{f_{j_1 j_2}^{\lambda}v_{j_1 j_2}^{(-)}}{\sqrt{\mathcal{Y}_\tau^{\lambda i}}} \begin{Bmatrix} \lambda_1 & \lambda_2 & \lambda \\ j_2 & j_1 & j_3 \end{Bmatrix} (\psi_{j_2 j_3}^{\lambda_2 i_2}\phi_{j_3 j_1}^{\lambda_1 i_1} + \psi_{j_3 j_1}^{\lambda_1 i_1}\phi_{j_2 j_3}^{\lambda_2 i_2}) +$$

$$\frac{f_{j_1 j_2}^{\lambda_1}v_{j_1 j_2}^{(-)}}{\sqrt{\mathcal{Y}_\tau^{\lambda_1 i_1}}} \begin{Bmatrix} \lambda_1 & \lambda_2 & \lambda \\ j_3 & j_2 & j_1 \end{Bmatrix} (\phi_{j_3 j_1}^{\lambda_2 i_2}\phi_{j_2 j_3}^{\lambda i} + \psi_{j_2 j_3}^{\lambda i}\psi_{j_3 j_1}^{\lambda_2 i_2}) +$$

$$\frac{f_{j_1 j_2}^{\lambda_2}v_{j_1 j_2}^{(-)}}{\sqrt{\mathcal{Y}_\tau^{\lambda_2 i_2}}} \begin{Bmatrix} \lambda_1 & \lambda_2 & \lambda \\ j_1 & j_3 & j_2 \end{Bmatrix} (\phi_{j_2 j_3}^{\lambda_1 i_1}\phi_{j_3 j_1}^{\lambda i} + \psi_{j_3 j_1}^{\lambda i}\psi_{j_2 j_3}^{\lambda_1 i_1})) \qquad (28)$$

and

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$$\Delta\omega^J(\lambda_1 i_1, \lambda_2 i_2) = -\frac{1}{4}\sum_{i\tau}[\frac{X_\tau^{\lambda_1 i_1} + X_\tau^{\lambda_1 i}}{\sqrt{\mathcal{Y}_\tau^{\lambda_1 i_1}\mathcal{Y}_\tau^{\lambda_1 i}}}K^J(\lambda_2 i_2, \lambda_1 i \mid \lambda_1 i_1, \lambda_2 i_2) +$$

$$\frac{X_\tau^{\lambda_2 i_2} + X_\tau^{\lambda_2 i}}{\sqrt{\mathcal{Y}_\tau^{\lambda_2 i_2}\mathcal{Y}_\tau^{\lambda_2 i}}}K^J(\lambda_2 i, \lambda_1 i_1 \mid \lambda_1 i_1, \lambda_2 i_2)] \tag{29}$$

Calculating the mean value of H we used so called quasidiagonal approximation for the quantities $K^J(\lambda_1 i_1, \lambda_2 i_2 \mid \lambda_3 i_3, \lambda_4 i_4)$ because the diagonal terms dominate over nondiagonal ones (see [20,21]).

Using the variational principle in the form:

$$\delta\langle\Psi_\nu(\lambda\mu)|H|\Psi_\nu(\lambda\mu)\rangle - E_\nu(\langle\Psi_\nu(\lambda\mu)|\Psi_\nu(\lambda\mu)\rangle - 1) = 0 \tag{30}$$

one obtains the following system of equations:

$$(\omega_{Ji} - E_\nu)R_i(J\nu) + \sum_{\lambda_1 i_1 \lambda_2 i_2} P_{\lambda_2 i_2}^{\lambda_1 i_1}(J\nu)U_{\lambda_2 i_2}^{\lambda_1 i_1}(J\nu)(1 + K^J(\lambda_1 i_1, \lambda_2 i_2)) = 0 \tag{31}$$

$$2(\omega_{\lambda_1 i_1} + \omega_{\lambda_2 i_2} + \Delta\omega^J(\lambda_1 i_1, \lambda_2 i_2) - E_\nu)P_{\lambda_2 i_2}^{\lambda_1 i_1}(J\nu) + \sum_i R_i(J\nu)U_{\lambda_2 i_2}^{\lambda_1 i_1}(Ji) = 0 \tag{32}$$

The energies of the states (23) are solutions of

$$F(E_\nu) \equiv det\left|(\omega_{\lambda i} - E_\nu)\delta_{ii'} - \frac{1}{2}\sum_{\lambda_1 i_1, \lambda_2 i_2}\frac{U_{\lambda_2 i_2}^{\lambda_1 i_1}(\lambda i)U_{\lambda_2 i_2}^{\lambda_1 i_1}(\lambda i')\left(1 + K^J(\lambda_1 i_1, \lambda_2 i_2)\right)}{\omega_{\lambda_1 i_1} + \omega_{\lambda_2 i_2} + \Delta\omega^J(\lambda_1 i_1, \lambda_2 i_2) - E_\nu}\right| = 0 \tag{33}$$

The rank of the determinant (33) is determined by the number of the one-phonon configurations included in the first term of the wave function (23).

The above derived equations have the same form as the basic QPM-equations [20,21] and we call them the extended QPM with the Pauli principle corrections (EQPMPP) in what follows. The GSC affect phonon energies $\omega_{\lambda i}$, normalization constants $\mathcal{Y}_\tau^{\lambda i}$ and renormalize the matrix elements of the quasiparticle-phonon interaction. If we put $K^J(\lambda_1 i_1, \lambda_2 i_2) = 0$ we get the equations already derived in[22] where all the fourth-order terms in phonon amplitudes $\psi_{jj'}^{\lambda i}$ and $\phi_{jj'}^{\lambda i}$ were neglected (we call this approach the extended QPM (EQPM)). In the limit $q_{jj'} \to 0$ one reproduces precisely all the expressions of the QPM with taking into account the Pauli principle corrections [20,21](QPMPP). In the case when $q_{jj'} = 0$ and $K^J(\lambda_1 i_1, \lambda_2 i_2) = 0$ we have equations describing coupling of one- and two- RPA phonons without taking into account the Pauli principle (QPM approach in following discussions).

3. Results and discussion

As an example we have performed numerical calculations for the ^{68}Zn. The Woods–Saxon potential parameters in use are from [23], slightly modified to better describe the ground state density. In so far as our single-particle spectrum includes the bound and quasibound states we do not use any effective charge to calculate the electromagnetic transition probabilities. The pairing constants $G_r^{(0)}$ are fixed so as to reproduce the odd-even mass difference of neighbouring nuclei. The strength parameters $\kappa^{(\lambda)}$ for the cases based on the RPA and ERPA schemes are adjusted so that the $B(E\lambda)$ values calculated with the wave function (23) within both approaches are reasonably close to the experimental ones. This means that $\kappa^{(\lambda)}$ for the ERPA calculations are larger than for the RPA ones [12]. No changes of $\kappa^{(\lambda)}$ have been done for calculations without the two-phonon terms. We use the ratio $\kappa_1^{(\lambda)}/\kappa_0^{(\lambda)} = -1.2$ that enables one to reproduce excitation energies of the isovector giant resonances in spherical nuclei. It is worth to mention that in our previous papers [11,12,23] we took into account only the isoscalar interaction for the p-h channel, but the inclusion of the isovector interaction does not affect the structure of low-lying states practically.

Our studies in Zn isotopes [12,23] of the effect of coupling of vibrations with different multipolarities (λ)(via $q_{jj'}$) show that in realistic calculations one can keep only λ=2 and 3 mainly. Solving nonlinear equations (9)-(14) one can find the phonon amplitudes,energies and the quasiparticle distributions within ERPA. Making use of them as input values it is possible to define from equations (24),(31)-(33) energies and the structure of the states described by the wave function (23).

Knowing the wave functions it is not difficult to calculate any matrix elements and physical quantities. For example, the charge transition density is calculated by the formula:

$$\rho_\nu^{(J)}(r) = \sum_{j_1 j_2} \rho_{j_1 j_2}^J(r)\{\frac{1}{2}(1 - q_{j_1 j_2})u_{j_1 j_2}^{(+)} \sum_i R_i(J\nu)(\psi_{j_1 j_2}^{Ji} + \phi_{j_1 j_2}^{Ji}) -$$

$$v_{j_1 j_2}^{(-)} \sum_{\lambda_1 i_1 \lambda_2 i_2} \sqrt{(2\lambda_1 + 1)(2\lambda_2 + 1)}P_{\lambda_2 i_2}^{\lambda_1 i_1}(J\nu)\sum_{j_3}(1 - q_{j_3 j_1}) \times$$

$$\begin{Bmatrix} \lambda_1 & \lambda_2 & J \\ j_1 & j_2 & j_3 \end{Bmatrix} (\psi_{j_2 j_3}^{\lambda_1 i_1}\phi_{j_3 j_1}^{\lambda_2 i_2} + \psi_{j_3 j_1}^{\lambda_2 i_2}\phi_{j_2 j_3}^{\lambda_1 i_1})\} \quad (34)$$

The expression for the two-quasiparticle transition density $\rho_{j_1 j_2}^J(r)$ can be found in[24]. Our charge transition densities are folded with the formfactor of the proton charge distribution [25]. Using quantities (34) one calculates the

reduced transition probabilities from the ground to the excited state $(J\nu)$[26]

$$B(EJ; 0^+ \rightarrow (J\nu)) = (2J+1) \mid \int\limits_0^\infty r^{J+2} \rho_\nu^{(J)}(r) dr \mid^2 . \qquad (35)$$

The results of our calculations for the quasiparticle distribution in the ground state of ^{68}Zn are shown in the table 1. It contains the values of q_j obtained in the RPA and ERPA schemes. As one can see from table 1 the q_j have large values for the subshells near the Fermi surface only and the ERPA gives stronger correlations in comparison with the RPA. A similar behaviour of q_j's has been found for other Zn isotopes [12]. This is valid for our choice of the multipole constants. As it was mentioned above the multipole constants $\kappa^{(\lambda)}$ have been chosen to describe with a reasonable accuracy experimental $B(E\lambda)$-values. The value of $\kappa_0^{(2)} = 0.0259$ MeV/fm for the non–linear problem is quite larger than the critical RPA constant $\kappa_0^{(2)} = 0.0242$ MeV/fm where the RPA solution becomes complex. (In the RPA case $\kappa_0^{(2)} = 0.0227$ MeV/fm). The octupole constant in use are equal to $\kappa_0^{(3)} = 0.0235$ MeV/fm for the RPA and $\kappa_0^{(3)} = 0.0250$ MeV/fm for the ERPA calculations respectively. The last value is smaller than the critical RPA constant for the octupole vibrations in contrast to the quadrupole ones. All the calculations based on the RPA phonons (RPA, QPM, QPMPP) have been performed with the RPA constants as well all the calculations based on the ERPA phonons (ERPA, EQPM, EQPMPP) have been done with the ERPA set of constants. It is worth to mention that as in the case of metallic clusters ([18]) our ERPA calculation gives also weaker correlations compared to the RPA ones, if the same set of multipole constants is used in both approximations, but in this case neither energies nor transition probabilities can be reproduced within the ERPA or its modifications.

To study the influence of the GSC on the quasiparticle-phonon coupling we calculated the structure of the low-lying states in ^{68}Zn with the wave function (23). Experimental data [27,28] and results of our calculations for the $2_{1,2}^+$ and 3_1^- states within different approaches are shown in the table 2.

One can see from table 2 that the RPA and ERPA overestimate the energies and fail to reproduce the transition probabilities for the 2_2^+ and 3_1^- states. Taking into account coupling of the one- and two-phonon components improves essentially the description of all states under consideration. Besides the transitions to the ground state, one can reproduce the B(E2)-value for the E2-transition between the first and the second 2^+ states. The inclusion of the Pauli principle corrections in two-phonon terms changes to worse the description of the second 2^+ state mainly. The EQPMPP describes energies and transition probabilities better than the QPMPP. It is worth to point out that the B(E2)-values for the E2-transition between the first and the second 2^+ states depend essentially on the two-phonon components of the wave func-

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tion of the 2_2^+ state and the last ones can be affected by the three-phonon terms, which are out of the present consideration. One can conclude that the most consistent approach from theoretical point of view (EQPMPP) where the Pauli principle is taken into account in both one- and two-phonon terms, gives a rather good description of experimental data in general.

The transition probabilities are the integral characteristics of the vibrational states and they are less sensitive to the details of the nuclear wave functions than the differential ones. As it was pointed out in our previous papers [11,12,23], GSC affect essentially the charge transition densities. Being the spatial overlap between the ground state wave function and the excited state wave function, the charge transition density provides a good test for nuclear models. The surface nature of the low-lying collective states predicted by calculations performed within the Hartree-Fock (HF) approach with effective forces [29] and the finite Fermi systems theory [30] has been demonstrated in the experiments on inelastic electron scattering from magic nuclei [31]. Experimental and theoretical (based on the random phase approximation (RPA)) studies of the charge transition densities [24,32] of the low-lying states in some spherical nuclei are in reasonable agreement, but the theory gives too large fluctuations of the transition densities in the interior region. All theoretical calculations in RPA, as in HF, give the same behaviour in the nuclear interior, which indicates a systematic problem of a more fundamental nature (a detailed discussion can be found in refs. [33,34]).

To test the developed approach we calculated the charge transition densities using eq.(34) in ^{68}Zn within different approaches. The figures 1 and 2 show the transition charge densities ($\rho_\nu^{(J)}(r)$) from the ground to the first 2^+ state in ^{68}Zn. The experimental data [28] are presented as a shadowed area. Fig.1 shows results of calculations based on the RPA phonons while calculations based on the ERPA phonons are presented in fig.2.

The RPA reproduces the behaviour of the charge transition densities qualitatively, but it overestimates the interior part of the $\rho_1^{(2)}(r)$. As one can see from fig.1, the inclusion of the two-phonon terms (see the QPM case) reduces the bump in the interior part of $\rho_1^{(2)}(r)$ by 17%. This is due to the reduction of the contribution of the first term in eq. (34) ($R_i < 1$). In spite of the dominance of the first term, the second one gives an additional reduction of ρ too. The Pauli principle correction for the two-phonon terms (QPMPP - case) change results not more than by 3%.

Taking into account GSC beyond the RPA results in a suppression of interior oscillations by 9% in comparison with the RPA case (see fig.2). Such a depletion is related with the Pauli blocking effect for the proton two-quasiparticle configuration $\{2p_{3/2}, 2p_{3/2}\}$, which is mainly responsible for the interior bump in the charge transition densities in the Zn isotopes. According to our RPA

calculations the proton two-quasiparticle configuration $\{2p_{3/2}, 2p_{3/2}\}$ gives a contribution about 40% into the norm of the first quadrupole phonon in ^{68}Zn. The inclusion of the GSC redistributes the strength of this configuration over many phonon roots and as a result the contribution into the first root becomes lower. The GSC suppresses the contribution of the partial proton two-quasiparticle $\{2p_{3/2}, 2p_{3/2}\}$ transition density for which $q_{2p_{3/2}}$ has the biggest value and as it was mentioned above plays an essential role in the structure of the interior part of the transition density for the 2_1^+ states. The configuration $\{1f_{5/2}, 1f_{5/2}\}$ gives some contribution to the interior part too and the same mechanism of suppression takes place for it. It should be noted that the amplitudes of the oscillations for the configurations with low orbital momenta are bigger than for the ones with high orbital momenta. That is because the single particle wave functions with low orbital momenta are mainly localized in the interior part of nuclei. Besides the blocking effect there are changes in coefficients $u_{j_1 j_2}^{(+)}$ because of the influence of the GSC on pairing and in the phonon amplitudes. All these effects suppress the interior bump. Taking into account the phonon coupling we get an additional lowering of the interior bump in the charge transition density and a reason of such lowering is the same as it was discussed above for the coupling of the RPA phonons. Finally the EQPM approach gives 30% reduction of the $\rho_1^{(2)}(r)$ in the interior nucleus region. It is worth to note that the Pauli principle corrections in two-phonon terms change the results slightly, but such corrections must be taken into account because they are often responsible for the weak electromagnetic transitions between excited states. One can see from eq.(35) that the outer part of the charge transition density is responsible mainly for the value of the reduced transition probability. Since all above discussed approaches have very close values for this part of the charge transition density, the calculated B(E2) are close too.

We would like to emphasize that the depletion effect discussed above can not be reproduced by any renormalization of the force strength $\kappa^{(2)}$ in the RPA if one wants to describe energies and reduced transition probabilities. An influence of the p-p channel of the multipole forces on the charge transition densities is under an investigation now. As concerning the charge transition density for the second 2^+ state, it is impossible to treat it without taking into account the three-phonon terms, which are out of the present consideration due to its numerical complexity.

The charge transition densities for the 3_1^- states in all Zn isotopes have a clear surface nature and there are no strong oscillations in the interior region of the nucleus because of a destructive interference of the 2-qp partial transition densities constructed from the single-particle wave functions with different parity.

4.Conclusion

A consistent treatment of the ground state correlations beyond the RPA including their influence on the pairing and phonon-phonon coupling is presented. A new general system nonlinear equations for the quasiparticle phonon model is derived. It is shown that this system contains as a particular case all equations derived for the QPM early. The system is solved numerically for first time in a realistic case for ^{68}Zn to study the effect of the GSC on the excitation energies, transition probabilities and charge transition densities of the vibrational states. Taking into account the GSC results in better agreement with experimental data for the characteristics of the low-lying states.

Up to now all realistic calculations taking into account GSC beyond the RPA have been performed with separable effective forces only (see [11,12,16,17]) because even for such simplified forces it is not very easy to solve the set of nonlinear equations in a large configuration space. The finite rank separable approximation for the Skyrme forces suggested recently [35] opens a new possibility to treat GSC beyond RPA basing on more realistic nuclear interactions.

5. Aknoweledgments

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Table 1
Quasiparticle distribution in the ground state of ^{68}Zn

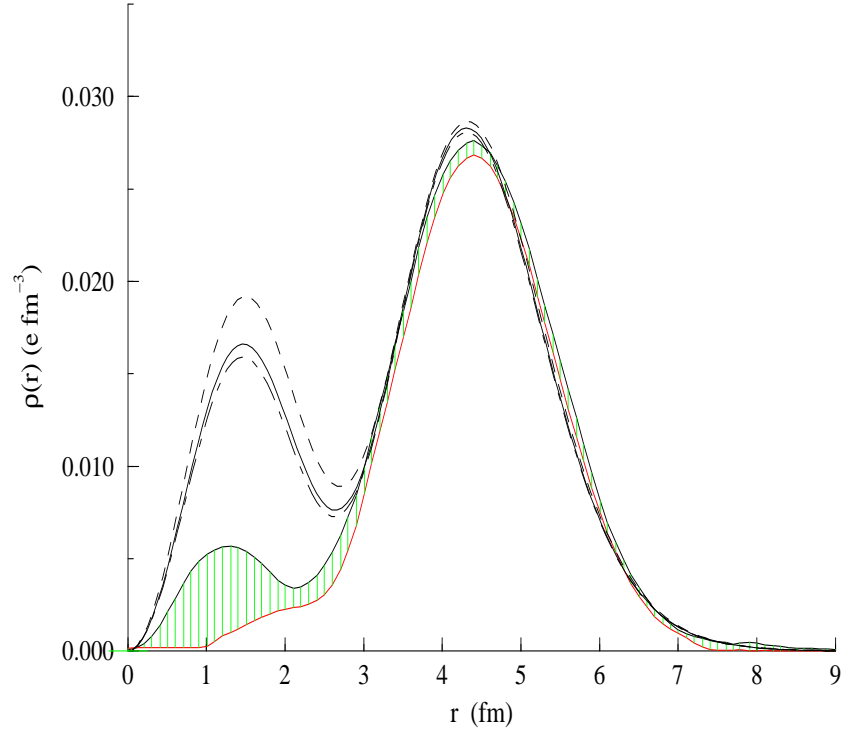| | q_{nlj} | | | |
| | neutrons | | protons | |
nlj	RPA	ERPA	RPA	ERPA
$2s_{1/2}$	0.0047	0.0134	0.0122	0.0189
$1f_{7/2}$	0.0163	0.0222	0.0320	0.0451
$2p_{3/2}$	0.0540	0.0651	0.0862	0.1053
$1f_{5/2}$	0.0482	0.0609	0.0324	0.0445
$2p_{1/2}$	0.0899	0.0994	0.0359	0.0461
$1g_{9/2}$	0.0230	0.0295	0.0144	0.0161
$2d_{5/2}$	0.0072	0.0087	0.0042	0.0047

Fig. 1. The transition charge density from the ground to the first 2^+ state in ^{68}Zn. The dashed line corresponds to the RPA result; the dot-dashed line - QPM; the full line - QPMPP. Experimental data are presented by a shadowed area.
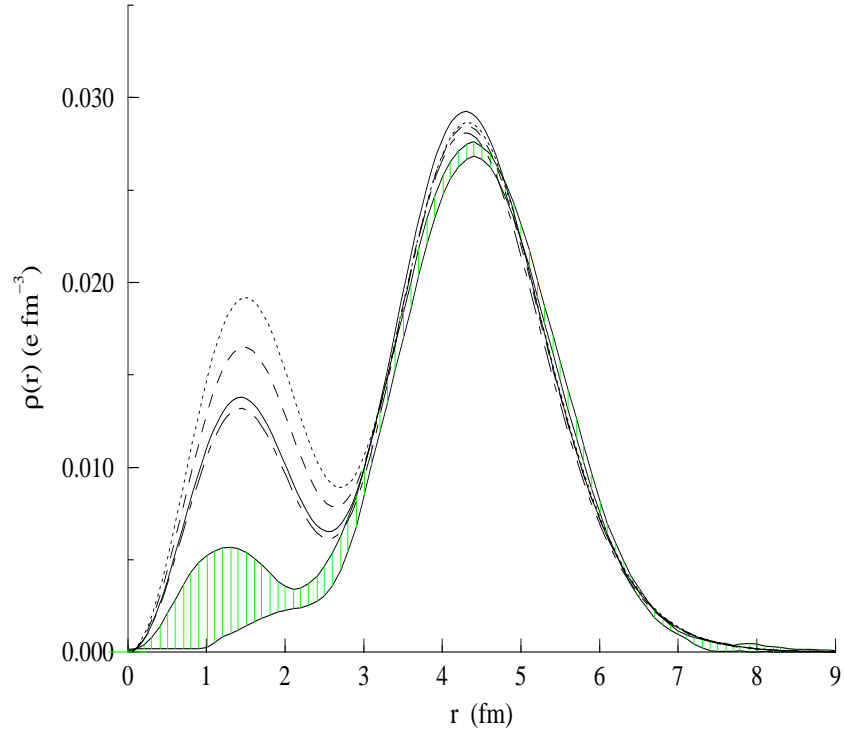
Fig. 2. The transition charge density from the ground to the first 2^+ state in ^{68}Zn. The dotted line corresponds to the RPA result; the dashed line - ERPA; the dot-dashed line - EQPM; the full line - EQPMPP. Experimental data are presented by a shadowed area.

Table 2
Energies and B(Eλ)-values for up-transitions to some first vibrational states of ^{68}Zn

State	2_1^+		2_2^+		3_1^-	
	Energy	B(E2)	Energy	B(E2)	Energy	B(E3)
		$0^+ \to 2_1^+$		$0^+ \to 2_2^+$		$0^+ \to 3_1^-$
				$2_1^+ \to 2_2^+$		
	(MeV)	(e^{2}fm^4)	(MeV)	(e^{2}fm^4)	(MeV)	(e^{2}fm^6)
EXP.	1.077	1266	1.883	46	2.751	38400
				287		
RPA	1.360	1290	2.390	75	3.830	48240
QPM	1.090	1200	2.060	48	2.760	38550
				231		
QPMPP	1.140	1220	2.180	27	2.840	39200
				351		
ERPA	1.330	1250	2.320	106	3.980	43070
EQPM	1.080	1170	1.810	47	2.760	34700
				212.		
EQPMPP	1.080	1270	1.960	38	2.750	35660
				436		